Mathematical Modelling for the Natural Sciences

EISI 2010

HJA Experimental Forest
“A mathematical model is a representation of the essential aspects of an existing system which presents knowledge of that system in usable form” [Eykhoff ‘74]

“The purpose of a model is to formulate a description of the mechanism in quantitative terms, and the analysis of it leads to results that can be tested against observations” [Fowler, ’97]

Mathematical modeling provides a systematic way of describing quantitatively and predicting the behavior of processes and systems.

[ -- Insert your own definition here -- ]
Components - Variables
Components - Variables
Components - Variables

Input
(don’t model)

Neglected
Components - Variables

Input
(don’t model)

Output
(model)
Components - Variables

Input
(don’t model)

Neglected

Mathematical relationships

Neglected

Output
(model)
Problem - Model - Solution
Problem - Model - Solution

Physical formulation
Problem - Model - Solution

Physical formulation → Related variables

Related variables → Solution

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Problem - Model - Solution

Physical formulation → Related variables → Mathematical formulation

→

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Problem - Model - Solution

Physical formulation → Related variables → Mathematical formulation

Solve for output
Problem - Model - Solution

- Physical formulation
- Related variables
- Mathematical formulation

- Physical solution
- Solve for output
Problem - Model - Solution

- Physical formulation
- Related variables
- Mathematical formulation
- Solve for output
- Physical solution
- Test against knowledge
Problem - Model - Solution

Physical formulation → Related variables → Mathematical formulation

Test against knowledge → Physical solution → Solve for output
Problem - Model - Solution

Natural sciences

- Physical formulation
  - Related variables
    - Mathematical formulation
      - Solve for output
        - Physical solution
          - Test against knowledge
Problem - Model - Solution

Natural sciences
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  - Test against knowledge
- Related variables
- Physical solution
- Mathematical formulation
  - Solve for output

Mathematics
Learning from models

'it is not possible to simultaneously maximize generality, realism and precision” [Levins]
Types of models

Specific
Data driven
Numerical
Stochastic
Microscopic
Discrete
Qualitative

Macroscopic
Quantitative
Conceptual
First principles
Deterministic
Analytical
Continuous
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Judging Models

There is no such thing as a correct model
Judging Models

There is no such thing as a **correct** model

- consistent
- accurate
- elegant
- optimal
- parsimonious
- robust

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There is no such thing as a correct model

consistent accurate
elegant optimal
parsimonious robust

The real question is:
Did we gain any useful understanding while making the model?
Some references

- Mathematical Methods for Engineers and Geoscientists By Olga Wälder, 2008
- An introduction to mathematical modeling By Edward A. Bender, 2000
- The nature of mathematical modeling By Neil A. Gershenfeld, 1999